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Metastable states of the random-field Ising chain

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Abstract. Using a numerical transfer-matrix method, we have determined the number of zero-temperature metastable states and their distribution in energy and magnetization for random-field Ising chains. Both discrete and continuous distributions of the random fields are considered. The degeneracy and structure of the metastable states and ground states for the discrete case may be understood in terms of a domain picture. Algorithms for generating exact ground-state configurations for both discrete and continuous distributions are described, and these demonstrate that the ground states of the random-field Ising chain have a hierarchical structure.

1. Introduction

Despite much effort since its introduction 15 years ago [1], many properties of the random-field model [2–4] are still not understood. Even the simplest model, the ferromagnetic Ising chain with random fields, has been incompletely explored, as the frustration renders the problem non-trivial even in one dimension. The random-field Ising chain is defined by the Hamiltonian

$$\mathcal{H} = -J\sum_i S_i S_{i+1} - \sum_i h_i S_i \tag{1}$$

where the spins take values $S_i = \pm 1$, the on-site external fields h_i are quenched variables chosen from a probability distribution $P(h)$, and the interactions $J > 0$ are ferromagnetic. While the first term in $\mathcal{H}$ favours ferromagnetic alignment of the spins, the second favours alignment with the on-site external field. The competition between these terms is the source of much of the complexity of the model; in particular, there exist exponentially large numbers of zero-temperature metastable states. As in the case of random-bond Ising spin glasses, the number N_s of these states grows with system size L as $N_s \sim 2^{\alpha L}$, where the value of α depends on the system. The large number of metastable states is linked to extremely slow relaxation at non-zero temperature.

The random-field Ising chain with discrete fields $h_i = \pm h$ (hereafter referred to as the '$\pm h$ model') has been studied extensively at zero temperature, either directly

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[5–7], or indirectly through investigations of the equivalent problem of the $\pm J$ spin glass (SG) chain in a uniform magnetic field h [8–10]. However, the zero-temperature metastable states of the random field Ising model (RFIM) chain have not been studied except indirectly through studies of the related problem of classical diffusion on a random chain [11, 12].

In the present paper we study the number of metastable states, and their distribution in energy and magnetization for both discrete (bimodal) and continuous (Gaussian and uniform) distributions $P(h)$ of the random fields. The numerical transfer-matrix method employed was originally developed for the Ising spin glass and is described in detail in a previous paper [13]. A chain consisting of L Ising spins has a total of 2^L possible states. The number of metastable states N_s is the subset of these states in which each spin S_i is aligned with the total magnetic field H_i acting on it, that is

$$N_s = \sum_{S_1}\sum_{S_2}\cdots\sum_{S_L}\prod_{i=1}^{L}\Theta(H_i S_i) \tag{2a}$$

where the unit step function Θ

$$\Theta(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases} \tag{2b}$$

expresses the constraint of metastability that the spin S_i must be aligned with the total field $H_i = h_i + JS_{i-1} + JS_{i+1}$ at site i. We define a 2×2 matrix $N_L(S_L, S_{L-1})$ which gives the number of metastable states of the chain of L spins for the four configurations of the last two spins, S_L and S_{L-1}. The matrix N_{L+1} for a chain of length $L+1$ is then obtained recursively as follows

$$N_{L+1}(S_{L+1}, S_L) = \sum_{S_{L-1}} N_L(S_L, S_{L-1})\Theta(H_L S_L). \tag{3a}$$

The matrix is initialized to have all elements equal and with their sum equal to unity. A value for the random field h_i is chosen from the distribution $P(h)$ and the new matrix is calculated. At each step the matrix is again normalized so that the sum of the elements is unity and the normalization factor a_i is extracted. The logarithm of the number of metastable states per spin is obtained by averaging these factors as follows

$$\frac{1}{N}\overline{\ln N_s} = \lim_{N\to\infty}\frac{1}{N}\sum_{i=1}^{N}\ln a_i. \tag{3b}$$

The distribution of metastable states as a function of the energy per spin (ε) and the magnetization per spin (m) is determined by inserting into (3a) the factor $\exp(\beta E_L + \beta_1 S_L)$, where $E_L = -JS_L S_{L+1} - h_L S_L$. This factor weights the contribution of each state according to its energy and magnetization. The parameters β and β_1 are Lagrange multipliers which determine the average energy and magnetization respectively. Recursion relations similar to (3a) for the derivatives of N_L with respect to (β_1, β) are easily obtained [13]. Thus, to each pair (β_1, β) correspond a magnetization m and an energy ε, and the sum of the matrix elements

$N_L(\beta_1, \beta) = \sum N_L(S_L, S_{L-1}; \beta_1, \beta)$ is averaged as in (3*b*) to yield the distribution of metastable states $(1/N)\overline{\ln N_s(m, \varepsilon)}$ as a function of the parameters (β_1, β), or equivalently (m, ε). Our method reproduces the exact results [5, 8] for the ground-state entropy and energy of the $\pm h$ model with an error of less than 1% for a chain of length 10^4 spins.

In the case of the Ising SG chain, the metastable states are simply obtained from the ground state by flipping clusters delimited by weak bonds [14], and so the domains defined by these clusters are easily identified. Likewise, the alternating sections of up and down spins in the metastable states of the RFIM chain can be obtained by overturning clusters, starting from a ferromagnetic state.

In section 2, we determine the number of metastable states and their distribution in energy and magnetization for the discrete $\pm h$ model with equal probability of positive and negative on-site fields. The denegeracy and structure of the metastable states and the ground states are explained, and an algorithm for finding the ground states is given.

In section 3, the distribution of metastable states for a random-field chain with the fields chosen from continuous distributions is investigated. As for the discrete case, the metastable states and ground states are understood in terms of a domain picture and a ground-state algorithm is also described.

2. $\pm h$ distribution

We consider first the simplest random field model, the ferromagnetic Ising chain with discrete random fields $+h$ or $-h$ $(h > 0)$ occurring with equal probability. Clearly, only values of h in the range $0 < h < 2J$ are of interest: for a large field $h > 2J$, all spins align in the direction of their site-random fields so that there is a unique 'metastable' state, while for $h = 0$, the problem reduces to the well known one-dimensional Ising model which has two degenerate ferromagnetic 'metastable' states. In this range of fields $(0 < h < 2J)$, all states which are metastable at one value of h are metastable at any other value, since only the signs of the random fields h_i are relevant to the metastability (to be shown later). Therefore, for the purpose of counting the number of metastable states, only a single magnitude h need be considered. However, the magnitude of h affects the energies of the states, and there are many energy crossings as h is varied, affecting the distribution $(1/N)\overline{\ln N_s(m, \varepsilon)}$.

In the metastable states of the $\pm h$ RFIM chain, a domain comprising ℓ up spins $S_{i+1}, \ldots, S_{i+\ell}$ all up, delimited by down spins $S_i, S_{i+\ell+1}$, can occur only where the random fields satisfy the following restrictions: $h_i < 0$, $h_{i+1} > 0$ and $h_{i+\ell} > 0$, $h_{i+\ell+1} < 0$. (For down domains, the inequalities are reversed.) This is because each of the two spins adjacent to a domain wall have a net bond contribution to the local field of zero, since their nearest-neighbour sites necessarily have spins pointing in opposite directions. Thus, the metastable states of the RFIM chain are characterized by the existence of ordered domains of opposite order parameter, domain walls possibly occurring where the random fields change sign.

The $\pm h$ model has well known peculiarities, for example, the unusual behaviour in the ground-state entropy and staggered magnetization [5, 8]. In particular, the discreteness of the random field distribution gives rise to a ground state highly degenerate at all values of h. Since the random field may take only the values $+h$ or $-h$, there are many positions of the domain walls which are energetically equivalent.

(See, for example, the degenerate configurations (*b*), (*c*) and (*d*) of figure 1.) This contributes to the degeneracy in the ground state at any value of the field h. The energy cost of inverting an arbitrary block of spins bounded by two unbroken bonds is $\delta E = +2(2J) - n(2h)$, where n is the net excess (positive n) or deficit (negative n) of spins aligned with their on-site fields in their final configuration. When $h = 2J/n$, this energy cost vanishes and the existence of certain well defined clusters which can be inverted with no energy cost leads to spikes in the ground-state entropy at fields $h = 2J/n$, where n is a positive integer. These ground-state degeneracies have previously been observed for the Ising chain with random exchange $\pm J$ [5] and for the Ising chain with random field $h_0 \bullet h$ [7]. Degeneracies also arise in the two-dimensional $\pm h$ RFIM and are associated with the existence of similar clusters in two dimensions [15, 16]. In the same way, energetically equivalent positions of the domain walls can occur for the metastable states of the $\pm h$ chain, giving rise to large numbers of degenerate metastable states.

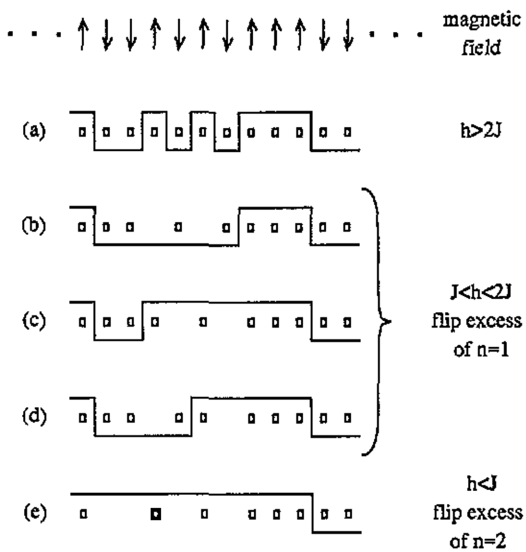


Figure 1. The ground states of the $\pm h$ RFIM chain are generated successively from ground states at higher strengths of the random fields. (See text for details.) The directions of the on-site random fields are indicated by the arrows at the top of the figure. The 'up' steps represent up spins and the 'down' steps down spins, while the vertical lines between them indicate the positions of the domain walls. Small rectangles have been drawn at positions corresponding to sites where the spin is aligned with its random field.

We now present a picture of the energetics of an Ising chain in a $\pm h$ random field which gives some understanding of the nature of the ground state, and why the degeneracy arises. A procedure for the construction of the ground states for arbitrary magnitude of random field is developed from this picture. A cluster picture similar to ours has been used to calculate explicitly the ground-state magnetization and residual entropy for a particular case [7].

The generation of ground states for decreasing magnitudes of the fields can be described as a 'smoothing' process, in which the domains of the ground states become successively larger with decreasing field as it becomes energetically favourable for larger domains with more spins aligned in the direction of the on-site random fields to be reversed or 'flipped'. This is best demonstrated by a simple example, as shown in figure 1 for the discrete model. Consider the sequence $\{h_i\}$ of random fields whose directions are represented by the arrows in figure 1(a). For field magnitude $h > 2J$ the ground-state configuration has all spins of the chain aligned with the on-site fields. As the magnitude of field is decreased such that $J < h < 2J$, it is energetically favourable for domains with a net excess of $n = 1$ spin aligned in the direction of the on-site fields to be reversed in the new ground state. In this case, this may happen in one of three ways (shown in figures 1(b), (c) and (d)), all of which have the same energy $(6h + 5J)$. Hence the ground-state entropy is large. For fields $\frac{2}{3}J < h < J$, domains with a net excess of $n = 2$ spins aligned with the on-site fields must be reversed in the new ground state. This leads to the ground-state configuration in figure 1(e), which may be generated by the same procedure from any of the three ground-state configurations at the previous step. In this particular example, the procedure can only be carried out to $n = 2$, since no domains with a net excess of $n = 3$ spins aligned with the on-site fields occur. It is seen that, in general, the ground states for field strength $2J/(n + 1) < h < 2J/n$ can be generated from the ones for $2J/n < h < 2J/(n - 1)$ by reversal of domains which have a net excess of n spins aligned with the on-site fields. In this manner, the ground states of the $\pm h$ RFIM chain form a '*hierarchy*' in which the ground states for successively lower field strengths are derived from the ground states for the previous range of field magnitudes by flipping clusters of increasing size. This implies that for small fields $h \ll 2J$, large-scale rearrangement of spins occurs in the ground-state configuration as h is varied, and that the average net magnetization of the ground state of the $\pm h$ RFIM chain will be zero for infinitesimally small fields. Thus the net magnetization in the ground state changes discontinuously from $m = 1$ for the non-random (zero field) Ising chain to $m = 0$ for the RFIM chain.

Although all the configurations in the hierarchy of ground states are metastable at all values of the magnitude of the field, not all metastable states are contained within the hierarchy. The hierarchy of ground states is comprised of those metastable states for which the field energy is a minimum for the same bond energy. In other words, a configuration of the ground-state hierarchy is a metastable configuration in which, for a given number of domain walls, the domain walls are positioned to minimize the field energy. Thus, the configurations contained in the hierarchy will be the minimum energy metastable states for a fixed number of domain walls. This also means that while a ground state for fields $2J/(n + 1) < h < 2J/n$ has no domains containing a net excess of n or fewer spins aligned with their on-site fields, the domains of a metastable state may contain an arbitrary number (a net excess or even a net depletion) of aligned spins. A similar picture of the ground states and metastable states of the $\pm h$ RFIM is expected to be valid for any dimension d for which the fully ferromagnetic state is unstable at zero temperature, namely for $d \leqslant 2$.

In the rest of this section, we give the results of numerical calculations for the $\pm h$ RFIM chain. Figure 2 shows the projection of the distribution $(1/N)\overline{\ln N_s(m, \varepsilon)}$ of metastable states onto the $m - \varepsilon$ plane for a chain of 10^5 spins with $h = 1.5J$, chosen to be well away from the anomalous fields $h = 2J/n$. Each triangle represents a pair of parameters (β_1, β) used in the algorithm and is located at the corresponding magnetization m and energy ε. The values of m and ε for which there are no metastable states are located in the region outside the boundary formed by the densely packed triangles. Figure 2 is representative of the metastable states of the $\pm h$ RFIM chain for any magnitude h in the range $0 < h < 2J$ (except perhaps $h = 2J/n$). For different magnitudes of the random fields, the energies are shifted and rescaled. An important feature of the function $(1/N)\overline{\ln N_s(m,\varepsilon)}$ is that it does not vanish smoothly everywhere on the boundary, but drops discontinuously to zero at both the ground-state energy $\varepsilon = \varepsilon_o$ and the maximum energy $\varepsilon = \varepsilon_{\max}$, indicating that both the ground state and the state of maximum energy are highly degenerate.

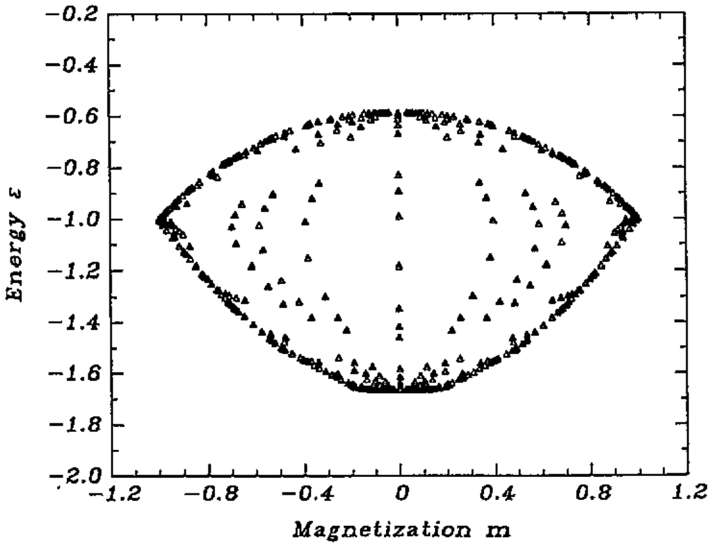


Figure 2. Projection of $(1/N)\overline{\ln N_s(m,\varepsilon)}$ onto the $m-\varepsilon$ plane for the $\pm h$ RFIM chain of length $L = 10^5$ with $h = 1.5J$. Each triangle represents a pair of parameters (β_1, β) used in the algorithm and is located at the corresponding magnetization m and energy ε.

The discontinuities at $\varepsilon = \varepsilon_o$ and $\varepsilon = \varepsilon_{\max}$ are shown in figure 3 which shows a cut through the function $(1/N)\overline{\ln N_s(m,\varepsilon)}$ on the $m = 0$ plane. The distribution is clearly asymmetric about the most probable energy. Figure 4 shows the faces of the function $(1/N)\overline{\ln N_s(m,\varepsilon)}$ at the planes corresponding to (i) the ground-state energy $\varepsilon = \varepsilon_o = -1.663$ and (ii) the maximum energy $\varepsilon = \varepsilon_{\max} = -0.5852$. There are exponentially many metastable states at both the ground-state and the maximum energies, the degeneracy being weaker at the latter. At both energies, the magnetizations are in the range $-0.15 \lesssim m \lesssim 0.15$.

3. Continuous distributions

For distributions with weights in both regions $|h| > 2J$ and $|h| < 2J$, there are clearly two kinds of sites. If $|h_i| > 2J$ the spin at site i is 'pinned' in the direction of the external field h_i, regardless of the configuration of its neighbours; if $|h_i| < 2J$ the spin can be either up or down, depending on the contribution ($-2J$, 0 or $2J$) from its neighbours to the total field at site i.

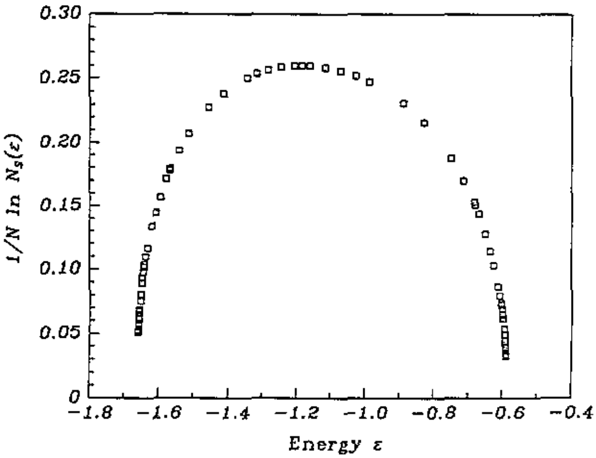


Figure 3. The distribution $(1/N)\overline{\ln N_s(\varepsilon)}$ of metastable states with respect to energy ε for the $\pm h$ RFIM chain with $h = 1.5J$.

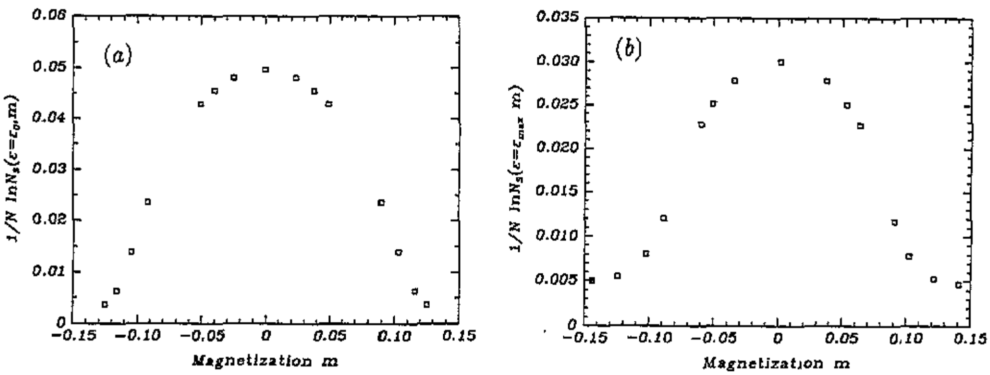


Figure 4. Cuts parallel to the m-axis: (a) a view of the plane through the function $(1/N)\overline{\ln N_s(m,\varepsilon)}$ at the ground-state energy, and (b) a similar plane through $(1/N)\overline{\ln N_s(m,\varepsilon)}$ at the maximum energy of the metastable states.

With respect to metastable states, the RFIM chain with random fields from continuous distributions can be considered as follows: spins at sites with strong ($|h| > 2J$) random fields are pinned, defining sections of the system containing only random fields of magnitude less than $2J$. Within these sections, domains necessarily consist of two or more spins. Since the exact magnitudes $|h_i|$ of the external random fields do not affect the metastability of the states, these sections are similar to finite-length $\pm h$ RFIM chains, but subject to special boundary conditions due to the pinned spins, which may have a non-local effect. (The effect of the strong fields becomes more evident for the ground state.) The pinning effect of strong fields extends beyond the single site on which it occurs only if the weak fields on sites adjacent to the pinning site are of the same sign as the strong field at the pinning site. In fact, the range of the pinning effect is limited to just those sites, and the rest of the system acts like independent $\pm h$ RFIM chain segments: domain walls within each segment occur where the random fields change sign, in a manner such that the orientation of the domains is consistent with the random fields (as in section 2).

From this simple structure of the metastable states it is seen that the details of the distribution, apart from the probability of strong fields, do not affect the total number of metastable states nor the magnetizations of the metastable states. These details affect the energy, however, and one obvious effect is that the ground state is non-degenerate.

An algorithm for generating the ground state of the RFIM chain with a continuous distribution $P(h)$ of random fields can be constructed by analogy with that for discrete distributions. Clearly, in the ground-state configuration the spins at sites where strong fields $|h_i| > 2J$ occur are 'pinned' in the direction of the fields. We consider then segments of the chain bounded by two consecutive strong fields such that except for spins at the first and last sites of the segment, all spins have weak fields $|h_i| < 2J$ acting on them. In order to incorporate the effect of the strong fields (which may extend beyond a single site) the segments thus defined must include the two bounding strong-field sites.

The configuration of a particular segment in the ground state may be explicitly constructed, and the ground state of the entire chain follows from joining the segments. For a given segment, the initial configuration is chosen such that every spin is aligned with the random-field direction. This defines ferromagnetically aligned clusters of all sizes (n-clusters, where $n = 1, 2, \ldots$) which may be sequentially reversed to generate the ground state.

The change in energy due to reversal of an n-cluster is

$$\Delta E = 2\left\{ -2J + \sum_{i=1}^{n} h_i S_i \right\}$$

where the sum is over the sites i in the cluster, from which we see that for any cluster with $\sum_{i=1}^{n} h_i S_i > 2J$, the spins of the cluster are aligned with their on-site fields in the ground state.

We then follow a sequential procedure similar to that of the previous section where we flipped clusters having a net excess n of spins aligned with the on-site fields when $h < 2J/n$, where n was an integer incremented at each step. The natural extension of this to continuous distributions is to make n a continuously varying increasing parameter. In the discrete case, an increasing n was associated with reversal of clusters with increasing net excess of spins aligned with their random fields. The analogue of this for the continuous case is the reversal of clusters with increasing net excess of field energy. Thus, the domain with the smallest energy cost less than $2J$ is reversed. Although accidental degeneracies may occur, their frequency does not increase exponentially with the number N of spins, and so may be neglected. Reversal of the first cluster creates a larger cluster by merging it with its two neighbouring clusters, so that the clusters must be redefined and the energy of the new cluster calculated. At each step, the cluster with the smallest random-field energy cost is reversed, and the clusters redefined.

The sequential procedure may be summarized as follows. (1) The initial clusters are defined by the configuration in which all spins are aligned with their fields. (2) The cluster with the *smallest* random-field energy cost less than $2J$ is reversed and (3) the clusters are redefined and the field energy of the new cluster calculated. Steps (2) and (3) are then repeated until no more clusters can be reversed without increasing the energy. In this way, we consider larger and larger domains, whose on-site field energies sum to less than $2J$ and which do not contain any sites with random field $|h_i| > 2J$. Thus the ground state contains large domains.

In fact, the domains are as large as possible, subject to the constraints imposed by the pinning of spins by strong ($|h_i| > 2J$) random fields, as well as by infrequent occurrences of consecutive fields of the same sign whose magnitudes sum to greater than $2J$. The probability that a domain of n spins is stable in the ground state is given by the probability that the sum of the n random fields is greater than $2J$. Since the random fields are independent, the probability distribution for the sum of n random fields is given by

$$P_n\left(z = \sum_{i=1}^{n} h_i\right) = \frac{1}{\sqrt{2\pi n}} \exp\left(-\frac{1}{2}\left(\frac{z}{\sqrt{n}}\right)^2\right) \tag{7}$$

where the width of the distribution increases as $\sqrt{n}$. The probability that the sum of n fields is greater than $2J$ is

$$2\int_{2J}^{\infty} \mathrm{d}z\, P_n(z) = \frac{2}{\sqrt{\pi}} \int_{\sqrt{2/n}\,J}^{\infty} \exp\left(-x^2\right) \mathrm{d}x \tag{8}$$

which increases with n. Hence, in the ground-state configuration, large domains are more likely to occur than small domains; small domains are a result of unlikely events. Of course, the actual size of the domains is governed by the constraints imposed by strong fields, and consecutive random fields in the same direction which have a non-negligible collective effect.

Since the ground state of the RFIM chain is strictly non-degenerate for the continuous distributions, the distribution of metastable states $(1/N)\overline{\ln N_s(\varepsilon)}$ goes to zero at $\varepsilon = \varepsilon_0$, as well as at $\varepsilon = \varepsilon_{\max}$, as in figure 5 which shows a typical density of metastable states curve for a Gaussian distribution of random fields with width $h = 0.75$. The distribution of metastable states with energy very closely resembles that for the random-bond Ising model with a continuous distribution of random bonds [13]. The energy spread of the distribution of metastable states decreases with the width of the distribution of the random field, as expected from the pinning picture.

The low-energy excitations of the RFIM chain are due to reversal of domains, and hence the behaviour at low energies should be similar to that of a set of weakly-coupled two-level systems, namely the expected density of metastable states at low energies should be given [14] by

$$(1/N)\overline{\ln N_s(\varepsilon - \varepsilon_o)} \sim (\varepsilon - \varepsilon_0)^{1/2}. \tag{9}$$

We find the exponent to be equal to $\frac{1}{2}$, within the accuracy of our numerical results.

Four cases of continuous distributions were considered, distinguished by type—either Gaussian or uniform and the width of the distribution—either wide or narrow. The projections for wide Gaussian and uniform distributions with the same integrated probability $\int_{-2J}^{+2J} P(h)\,\mathrm{d}h = 0.4323$ are shown in figures 6(*a*) and 6(*b*), respectively. Figures 7(*a*) and 7(*b*) are the projections of the corresponding narrow distributions with $\int_{-2J}^{+2J} P(h)\,\mathrm{d}h = 0.9923$.

Typically, the projection of the distribution of metastable states $(1/N)\overline{\ln N_s(m,\varepsilon)}$ onto the $m-\varepsilon$ plane for continuous distributions has a shape which tapers off toward

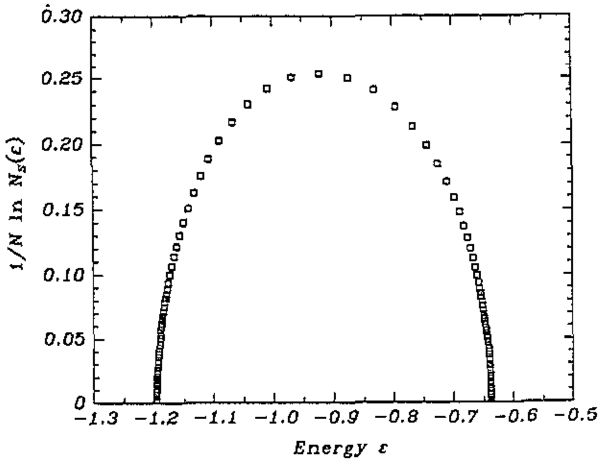


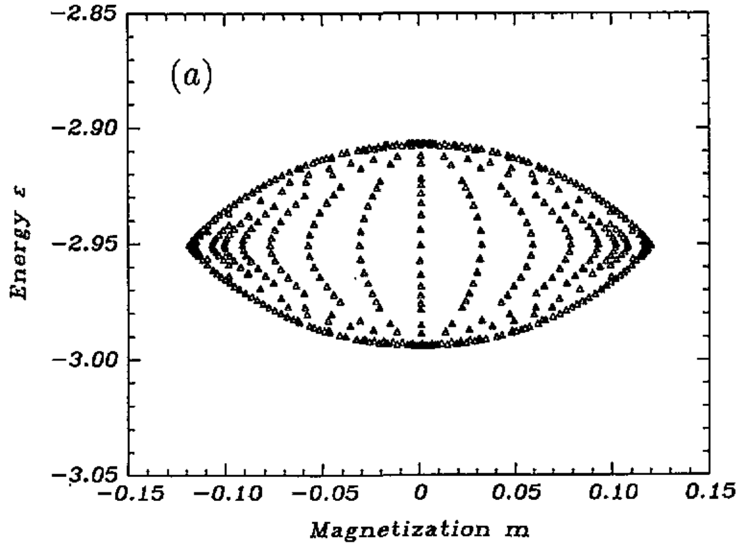
Figure 5. A typical density of metastable states for a Gaussian distribution of random fields with width $h = 0.75J$.

$\pm m_{\max}$ (see figures 6 and 7). Again, the total number of metastable states and the maximal magnetization of the metastable states depends only on the total probability of fields having magnitude greater than $2J$, and only the energies rescale and shift with varying distributions of random fields. The maximal magnetization $m_{\max}$ of the metastable states must be less than $1 - \int_{2J}^{\infty} P(h)\,\mathrm{d}h$; this represents an upper limit since strong fields whose effect extends to spins at adjacent sites and strings of sites where random fields all of the same sign sum to $|\sum_{i=1}^{n} h_i| > 2J$ both contribute to reducing the magnetization of the maximally magnetized state.

The shift in energy of figures 6(a) and 6(b) is primarily due to the field energy of fields of magnitude greater than $2J$. This can be seen clearly from the energy of the state with maximal magnetization, that is the state with all spins up except for those which are constrained to be in the opposite direction by single strong down fields, or clusters of down fields. The energy of such a state is approximately:

$$E = E_{\uparrow} + 2 \times \frac{2J}{N} \text{ (no of down domains) } - 2 \int_{-\infty}^{-2J} P(h) h \,\mathrm{d}h \qquad (10)$$

where $E_{\uparrow} = -J + \int_{-\infty}^{\infty} P(h) h \,\mathrm{d}h = -J + O(N^{-1/2})$ is the energy per spin of the state with all spins up, and the contributions to the field energy from spins aligned with small ($h < 2J$) fields have been neglected. Since the number of down domains in the state of maximal magnetization depends strongly on the total probability of fields less than $-2J$ (this number also depends on the details of the distribution through the probability that consecutive fields have the same sign and magnitudes which add up to greater than $2J$), the energy due to domain walls should be roughly the same as long as these probabilities are the same. Thus the difference in energy between the maximally magnetized metastable states for two symmetric random-field distributions with the same area $\int_{-2J}^{2J} P(h)\,\mathrm{d}h$ is largely due to the difference in field energy of random fields with magnitude greater than $2J$ pointing in the direction opposite to the magnetization. Then the shift in energy for the wide distributions of figure 6 is roughly



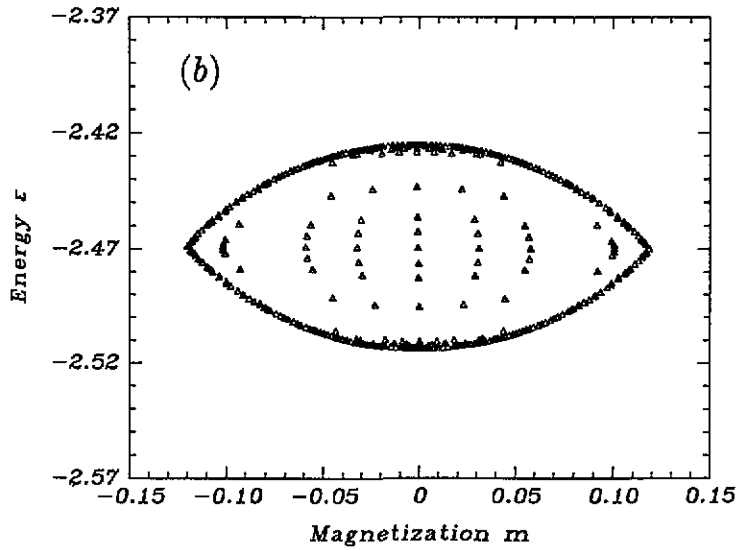


Figure 6. Projection of $(1/N)\overline{\ln N_s(m,\varepsilon)}$ onto the $m-\varepsilon$ plane for wide distributions of random fields: (*a*) Gaussian and (*b*) uniform. Each triangle represents a pair of parameters (β_1, β) used in the algorithm and is located at the corresponding magnetization m and energy ε.

$$
\begin{aligned}
\delta E &\simeq E_h(\text{Gauss}) - E_h(\text{uniform}) \\
&= -2\int_{2J}^{\infty} P_2(h)h\,\mathrm{d}h + 2\int_{2J}^{\infty} P_1(h)h\,\mathrm{d}h \\
&= -2.372 - (-1.881) = -0.49
\end{aligned}
$$

which is not too far from the actual shift of $\delta E = -0.48$. This picture breaks down for the narrow distributions. Since there are few pinned spins, the metastable state of maximum magnetization is almost ferromagnetic, so that its energy is nearly $-J$. The energies of the metastable states are no longer just shifted, but depend strongly

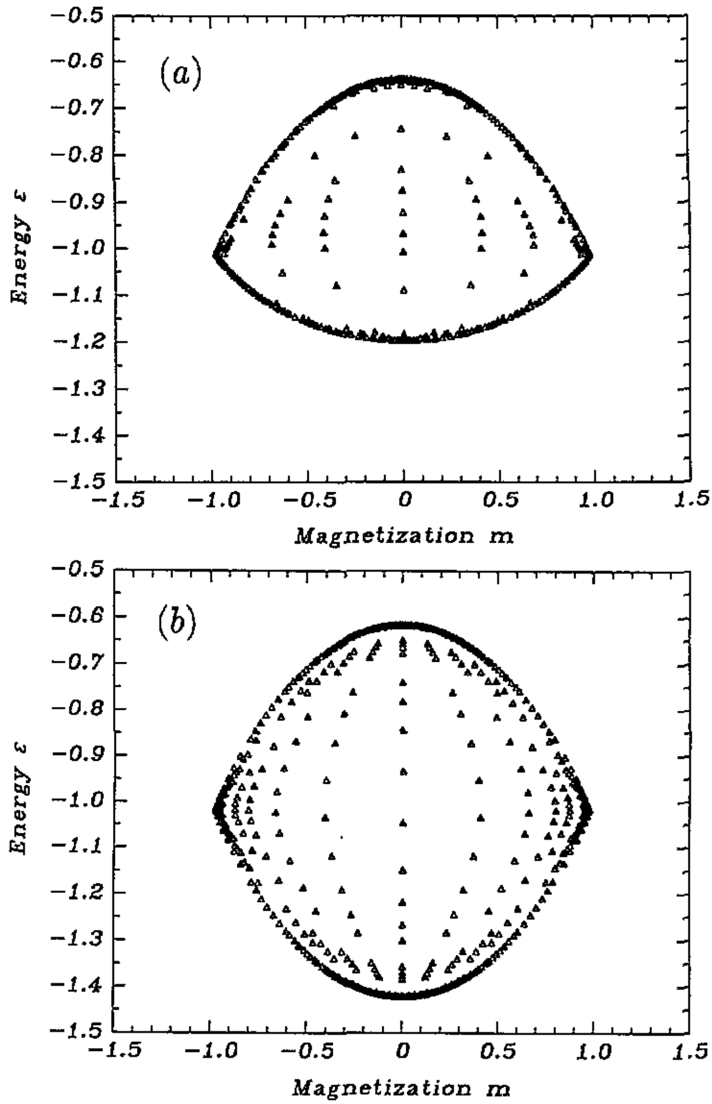


Figure 7. Projection of $(1/N)\overline{\ln N_s(m,\varepsilon)}$ onto the $m-\varepsilon$ plane for narrow distributions of random fields: (*a*) Gaussian and (*b*) uniform. Each triangle represents a pair of parameters (β_1, β) used in the algorithm and is located at the corresponding magnetization m and energy ε.

on the random-field distribution since the narrow distributions differ considerably in the region $-2J < h < 2J$, while the wide distributions were sufficiently similar in this region.

4. Summary

We have determined the number of metastable states and their distribution in energy and magnetization for RFIM chains with a discrete (bimodal) distribution, and with two continuous distributions: Gaussian and uniform. The degeneracy and structure of the metastable states and ground states may be understood in terms of a cluster picture. Algorithms for generating exact ground-state configurations for both discrete and continuous distributions have demonstrated that the ground states of the RFIM chain have a hierarchical structure.

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